then, as Hurwitz showed,

$$(10) \qquad \beta_{4m} = (1 + i)^{4m}\{(1 + i)^{4m} - 2\}E_m .$$

The writer [1, Theorem 9] has proved that if $p^r \mid m$, $p - 1 \nmid 4m$, then

$$\beta_{4m} \equiv 0 \pmod{p^r}.$$

Thus (10) yields

$$(11) \qquad \{(1 + i)^{4m} - 2\}E_m \equiv 0 \pmod{p^r}.$$

Since $p - 1 \nmid 4m$, the denominator of E_m is not divisible by p, so that (11) implies

$$(12) \qquad \{(1 + i)^{4m} - 2\}N_m \equiv 0 \pmod{p^r}.$$

In the next place, since

$$(1 + i)^{4m} = (-4)^m,$$

it is evident that

$$(1 + i)^{2m} - 2 \equiv 0 \pmod{p}$$

if and only if

$$(13) \qquad (-1)^m 2^{2m-1} \equiv 1 \pmod{p}.$$

If, therefore, (13) is not satisfied, it is clear from (12) that

$$N_m \equiv 0 \pmod{p^r}.$$

Finally we note that if $p = m$, then (13) is not satisfied and (7) follows at once.

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All Factors $q < 10^8$ in All Mersenne Numbers $2^p - 1$, p Prime $< 10^4$

By H. Riesel

During the year 1960 the author made additional investigations respecting factors of Mersenne numbers $M_p = 2^p - 1$, where p is a prime. The author has earlier examined the least factor q of M_p, for $p < 10^4$, if $q < 10 \cdot 2^{20} \approx 10^7$. This inquiry was secondary to an effort to ascertain Mersenne primes [1].

The present examination, which resulted in 255 new factors of the numbers M_p, was done in the following manner: All primes $s < 10^8$, for which $s \equiv 1 \pmod{2p}$ and $s \equiv 1 \pm \pmod{8}$ were tested as factors of M_p for $1200 < p < 10000$. The range $p < 1200$ has been examined by Brillhart and Johnson [2].

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To save computing time the following scheme was used. If it takes long to decide whether a number s of the kind in question is a prime, but an examination of whether $2^p \equiv 1 \pmod{s}$ can be accomplished in a short time, it may be convenient to test unnecessarily some s-values which are not primes. Such values can be obtained by using such s-values as do not contain any factor $\leqq t$, where t is given. If $\sqrt{s} \leqq t < s$ only primes remain. Generally, if $t < \sqrt{s}$ the fraction

$$\gamma = \gamma(t) = \prod_{p=3}^{t} \left(1 - \frac{1}{p}\right), \qquad p \text{ prime}$$

of all numbers remaining after this test. Preparing the program for use on the BESK, it was found by test runs that the computing time was minimal with the following t-values:

$$\begin{array}{lll} t = 40 & \text{for} & 1200 < p < 2000 \\ t = 70 & \text{for} & 2000 < p < 3000 \\ t = 90 & \text{for} & 3000 < p < 4000 \\ t = 120 & \text{for} & 4000 < p < 5000 \\ t = 130 & \text{for} & 5000 < p < 6000 \\ t = 170 & \text{for} & 6000 < p < 7000 \\ t = 200 & \text{for} & 7000 < p < 10000 \end{array}$$

The running time on the BESK was about 20 minutes for each p.

All Mersenne numbers M_p with $p < 10000$ have now been examined with respect to all factors $q < G = 10^8$; for some M_p this limit G has been considerably raised. A summary is given below of what in the author's opinion has so far been accomplished in this field:

$p \leqq 97$	completely factored
$p = 101$	$G = 2^{35}$
$103 \leqq p \leqq 149$	$G = 2^{31}$
$p = 151$	completely factored
$157 \leqq p \leqq 257$	$G = 2^{31}$
$263 \leqq p \leqq 389$	$G = 2^{30}$
$p = 397$	$G = 2^{32}$
$401 \leqq p \leqq 1021$	$G = 2^{30}$
$1031 \leqq p \leqq 1193$	$G = 2^{28}$
$1201 \leqq p \leqq 2999$	$G = 10^8$
$3001 \leqq p \leqq 3593$	$G = 10^8$, if any factor is known
$3001 \leqq p \leqq 3593$	$G = 38400p + 1$, if no factor is known
$3607 \leqq p \leqq 9973$	$G = 10^8$

All hitherto known factors are to be found in [1], [2] and this paper. Furthermore, the author has calculated the two largest factors of M_{97} and M_{151} ; these are

13842607235828485645766393 and 7289088383388253664437433

respectively.

As has earlier been pointed out [3], two typographical errors occurred in [1]; the factors belonging to M_{2689} and M_{5743} should be 7158119 and 643217, respectively.

Some of the factors referred to below have been calculated by Edgar Karst [4],

[5], namely, the 24 factors designated by an asterisk(*). Furthermore, all factors stated in [1] and [2] as well as in this paper have been checked with a special program and have been found correct, except for the typographical errors mentioned above.

Lastly, the author desires to express his gratitude to the Swedish Board for Computing Machinery, which has graciously provided machine time on the BESK for this study.

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EDITOR'S NOTE: During the period in which this paper was in process of publication A. Hurwitz [6] has listed 92 Mersenne composites, corresponding to primes p in the interval $330 < p < 5000$, for which no factor is known at present. He also lists M_{8191} as such a composite number, and he announces the primality of M_{4253} and M_{4423}. S. Kravitz [7] has extended the writer's earlier table [1] to the range $10{,}000 < p < 15{,}000$.

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p	Factors of M_p
1201	1967239·8510287
1223	31799
1229	46703
1231	5684759
1249	52358081
1321	1394977·4848071
1361	3397057
1367	65617·232391·2561759
1399	28875361
1433	53101249·86576129
1439	46049·172681
1543	472159
1583	189961·8589359·43817441
1627	25722871
1637	81679753
1663	6013409
1667	2493833·10975529
1723	56421359
1741	4230631
1789	254039
1861	2735671
1879	13697911
1901	12147391
1931	50207
1979	32392273
2063	53639·165041
2069	1816583
2081	3308791·3874823·7920287
2099	22858111
2113	52030513
2131	17359127
2251	778847*
2309	61636447
2339	299393·1445503
2389	172009·267569
2393	10758929
2399	5201033
2411	1393559·4185497
2579	19868617
2593	62233
2609	7404343
2617	2213983
2621	12895321
2657	148793·318841
2663	15743657
2677	465799
2687	1918519·2434423
2689	90092257
2699	307687·1187561
2711	76954447
2731	93968249
2749	45737863

p	Factors of M_p
2843	187639
2903	60457879
2917	22869281
2963	1374833·68747527
2999	95278231
3037	145777*
3041	5565031*
3067	22063999*
3083	15914447*·68886553
3119	230807*·14222641*
3121	31509617*
3167	12237289*
3181	127241*
3191	40895857*
3253	46452841*
3257	4032167*
3299	19873177*
3329	665801*·1005359*·26225863*
3391	1519169*
3433	5952823*·12688369*
3529	5335849*·13523129*
3557	2952311*
3593	6086543*
3701	318287·1561823
3719	9490889
3761	55655279
3767	50455199
3779	2086009·6084191
3793	91033
3863	1120271
3923	2236111
4003	16756559
4021	10800407
4099	262337
4127	2080009
4139	16820897
4211	92271433·94528529
4373	104953
4507	21381209
4597	73553·93769607
4759	161807
4903	13973551
4919	2872697
4931	33264527
4943	128519
4957	3003943
5003	1050631
5009	36425449
5087	1678711
5119	82978991
5189	3362473
5197	20195543
5227	47126633
5231	41849·470791
5273	38134337
5281	24715081
5303	6024209·33737687
5381	48170713
5417	3196031
5431	1401199
5507	21675553
5521	3533441
5563	623057
5569	37813511
5639	2447327
5717	45737
5741	48224401
5821	88805177
5827	91658711
5867	16239857
5903	15489473·42182839
6067	3118439
6121	81237913
6199	743881
6271	13959247
6287	5180489·57186553
6311	68423863
6317	4548241
6353	2172727
6373	140207
6427	10951609
6481	2799793·38678609
6491	69440719
6529	1201337
6563	26777041
6761	26340857
6841	64962137·77973719
6871	39027281
6947	333457
6977	209311
6983	13630817
7079	56633·1486591
7159	26058761
7187	48871601
7211	57689·475927
7253	52613263
7349	42197959
7451	95506919
7457	19969847
7499	99121783
7517	12012167
7547	16980751
7673	13918823
7717	19446841
7841	6837353
7883	441449
7901	442457·679487
8087	96671999
8111	41917649
8123	25798649

p	*Factors of* M_p	p	*Factors of* M_p
8167	76835137	9049	28721527 · 28938703
8171	9412993	9059	30293297
8209	14759783	9109	49625833
8221	9667897 · 18480809	9127	8707159
8269	19630607	9137	2704553
8273	28062017 · 62014409	9161	86901247
8287	36877151	9199	53354201
8377	134033 · 787439 · 2596871	9221	91841161
8429	455167 · 927191	9283	29352847 · 34031479 · 41532143
8467	6655063	9337	2838449 · 2405633
8539	13662401	9403	5735831
8563	32402393	9479	48532481
8573	12345121	9601	3513967 · 16974569 · 17256487
8623	80504329	9643	12362327
8699	43790767	9743	34626623
8737	6640121	9817	20556799
8741	5926399	9829	14075129
8849	52368383	9851	3723679
8933	36232249	9859	1656313
8969	13345873	9883	10436449
9029	25913231	9973	7419913 · 10591327 · 19367567

Certain Properties of Pyramidal and Figurate Numbers

By M. Wunderlich

It is well known that despite some extensive computation [1], the only two known solutions to the Diophantine equation

$$(1) \qquad a^3 + b^3 + c^3 = 3$$

are $a = b = c = 1$, and $a = b = 4$, $c = -5$. Professor Aubrey Kempner noted at a number theory seminar at the University of Colorado that these solutions also satisfy the equation

$$(2) \qquad a^3 + b^3 + c^3 = a + b + c.$$

Therefore, it is of interest whether or not (2) has solutions other than these two and if so, how many. Since there are so few solutions known to (1), it seemed reasonable to conjecture that there would be only finitely many solutions to (2).

If we change the sign of the third variable and divide through by six, we see that (2) is equivalent to

$$\frac{a^3 - a}{6} + \frac{b^3 - b}{6} = \frac{c^3 - c}{6}$$

or

$$\frac{(a-1)(a)(a+1)}{6} + \frac{(b-1)(b)(b+1)}{6} = \frac{(c-1)(c)(c+1)}{6}.$$

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